parameters in this example are such[12] that third-order reorientation effects are small.

In Fig. 3 we have plotted $P(1,2)/P(1,1)$ as a function of the bombarding energy E for the fixed angle $\theta=180°$ using again the example of Cd^{114} with $Q=0.5\times10^{-24}$ cm^2 and $|\eta|=1$. The assumed quadrupole moment is probably quite realistic, so that both second-order effects may have the same size. The difference in the energy dependence (Fig. 3) or the difference in the angular behavior (Fig. 2) might serve to distinguish between the two effects.

Summarizing, we may say that any attempt to determine a quadrupole moment by the reorientation effect must take into account virtual transitions via the giant dipole resonance. This requires a higher experimental accuracy, but, on the other hand, a determination of the structure parameter η is an interesting problem in itself.

ACKNOWLEDGMENTS

The author would like to thank Dr. J. de Boer for valuable discussions which first drew his interest to Coulomb excitation. He furthermore wants to acknowledge the kind hospitality of the California Institute of Technology.

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[12] D. L. Lin and J. F. Masso, Proceedings of the Conference on Reactions between Complex Nuclei, Asilomar, California, 1963 (unpublished).

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Recoil Study of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ Reaction*

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The excitation function and the product recoil behavior of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction was studied using incident protons of energy between 80 and 430 MeV. The thick-target thick-catcher technique was used in which effective recoil ranges were measured in the forward, backward, and transverse directions. The data were interpreted in terms of the knock-out mechanism. The data were also fitted to a recoil velocity distribution written in terms of a power series in the cosine of the scattering angle. Ranges calculated by this treatment are consistent with the interpretation that the reaction proceeds mainly by the knock-out mechanism. Reasonable agreement was obtained between the recoil kinetic energy, calculated on the basis of the assumed recoil velocity distribution, and that which would be obtained from an abrupt removal of a proton from the top of the nuclear well in Zn^{68}.

INTRODUCTION

THAT class of high-energy nuclear reactions known as the "simple reactions" are thought to involve the interaction of the incident particle with the target nucleus via nucleon-nucleon collisions within the nucleus. For $(p,2p)$ and (p,pn) reactions, only one collision of the incident proton with the appropriate nucleon is required. In principle, if it can be assumed that the interaction involved in these simple reactions involves only the collision of the incident proton with a target nucleon, no other effects manifesting themselves, then an observation of the momentum distribution of the products should reflect the momentum distribution of the struck nucleons prior to the collision. Several groups of experimenters have measured the angular and energy distribution of the protons emitted in $(p,2p)$ reactions,[1–3] but no recoil distribution of the product nucleus has been measured at incident energies above 100 MeV. Most studies of $(p,2$ nucleon$)$ reactions have been confined to measurements of the cross section for the reaction as a function of the bombarding protons.[4–9] Only a few product recoil studies of (p,pn) reactions have been reported,[10–13] and of these only the $\mathrm{C}^{12}(p,pn)\mathrm{C}^{11}$ reaction, as studied by Singh and Alexander, and the $\mathrm{Cu}^{65}(p,pn)\mathrm{Cu}^{64}$ reaction, as studied by Merz and Caretto, were investigated in sufficient detail to examine the assumptions underlying the concepts of simple high-energy reactions. As an extension of these studies, preliminary recoil studies have been made of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction since (i)

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[1] H. Tyren, P. Hillman, and Th. A. J. Maris, Nucl. Phys. **7**, 10 (1958).

[2] Th. A. J. Maris, P. Hillman, and H. Tyren, Nucl. Phys. **7**, 1 (1958).

[3] T. J. Gooding and H. G. Pugh, Nucl. Phys. **18**, 46 (1960).

[4] S. Markowitz, F. S. Rowland, and G. Friedlander, Phys. Rev. **112**, 1295 (1958).

[5] H. P. Yule and A. Turkevich, Phys. Rev. **118**, 1591 (1960).

[6] N. T. Porile, Phys. Rev. **125**, 1379 (1962).

[7] P. P. Strohal and A. A. Caretto, Jr., Phys. Rev. **121**, 1815 (1961).

[8] N. T. Porile and S. Tanaka, Phys. Rev. **130**, 1541 (1963).

[9] L. P. Remsberg and J. M. Miller, Phys. Rev. **130**, 2069 (1963).

[10] N. M. Hintz, Phys. Rev. **86**, 1042 (1952).

[11] E. R. Merz and A. A. Caretto, Jr., Phys. Rev. **126**, 1173 (1962).

[12] S. Singh and J. M. Alexander, Phys. Rev. **128**, 711 (1962).

[13] N. I. Borisova, M. Ya. Kuznetsova, L. N. Kurchatova, V. N. Mekhedov, and L. V. Chistyakov, Zh. Eksperim. i Teor. Fiz. **37**, 366 (1959) [English transl.: Soviet Phys.—JETP **10**, 261 (1960)].

to date there are no product recoil studies of $(p,2p)$ reactions, (ii) it is of interest to compare the results of this study with those of the $\mathrm{Cu}^{65}(p,pn)\mathrm{Cu}^{64}$ reaction, which comparison should be valid due to the similarity of target mass number, (iii) the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction might be expected to be less complicated (i.e., take place by a nearly pure knock-out mechanism) than the $\mathrm{Cu}^{65}(p,pn)\mathrm{Cu}^{64}$ reaction and hence interpretation might be less ambiguous, and (iv) necessary range-energy data exist for masses 65–70 so that the Cu^{67} recoil ranges may be converted to the corresponding recoil kinetic energies. In regard to item (iii), the results of a more ambitious experimental technique involving the measurement of the 2π-differential range of the $\mathrm{Cu}^{65}(p,pn)\mathrm{Cu}^{64}$ reaction are presented in the following paper.[14]

EXPERIMENTAL PROCEDURE AND RESULTS

The targets for the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ excitation function and recoil study were prepared by electroplating enriched zinc, 96.8% Zn^{68}, onto 1-mil 99.999% gold foil. The zinc was obtained as ZnO from Oak Ridge National Laboratory and was electroplated by a method described by Exner.[15] A target thickness of 3–7 $\mathrm{mg/cm^2}$ was obtained in this manner.

The zinc-plated gold foil was placed on top of two pieces of 1-mil aluminum foil (Alcoa-1199), the gold adjacent to the aluminum, and an area of 0.25 ×0.25 in. was punched with a square punch. The aluminum foil next to the gold served as a guard foil, while the other was used as a monitor foil. This target-monitor stack was placed in the center of a 0.50-×0.50-×0.001-in. aluminum foil, which served as a catcher. A second aluminum foil of the same size, which served as a blank, was placed behind the catcher. The catcher foil was made larger than the target to prevent loss of any of the recoils near the edge of the target. All foils were wrapped in an aluminum envelope for mounting onto a probe for bombardment with the internal beam of the Carnegie Institute of Technology proton synchrocyclotron.

After bombardment, the zinc target was dissolved from the gold backing with 9*M* HCl and copper was separated chemically by the procedure reported by Merz and Caretto.[11] Prior to the first cuprous thiocyanate precipitation, a gold scavenge was done by bubbling $\mathrm{SO_2}$ through the solution. The samples were mounted as CuS on filter paper disks with a filter-chimney and the beta decay of the Cu^{67} was detected with a methane-flow end-window beta-proportional counter.

The effect of Cu^{67} nuclei produced by fission of gold on the recoil measurements of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction was estimated by bombarding an unplated gold foil in a stack of 0.3-mil aluminum foils. If the stopping power for heavy fragments is assumed to vary as the square root of the mass number of the medium, one 0.3-mil Al foil would correspond to a thickness of about 3 $\mathrm{mg/cm^2}$ of zinc. The fraction of Cu^{67} activity observed through this thickness was 2×10^{-3}, 2×10^{-3} and 3×10^{-3} in the forward, backward, and perpendicular directions, respectively, at a bombarding energy of 400 MeV. To find the relative uncertainty in the $(p,2p)$ data, these fractions from the fission of gold must be corrected for the magnitude of the fission cross section compared to the $(p,2p)$ cross section. By taking a value for the cross section for the production of Cu^{67} from gold of 0.44 mb,[16] the maximum contributions expected from Cu^{67} produced by fission to that produced in the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction in the recoil activities is 2% in the forward fraction, 5% in the perpendicular fractions and 10% in the backward fraction. Since these contributions are of the order of the uncertainty of the experimental results, they have been neglected.

Several runs were made at 130, 200, 400, and 430 MeV with natural zinc foil. The fraction of recoiling fragments observed in these experiments was within experimental uncertainty the same as that observed in the bombardments with the electroplated enriched Zn^{68}. There was apparently no significant contribution to the recoil activity from the $\mathrm{Zn}^{70}(p,2p2n)\mathrm{Cu}^{67}$ reaction.

Absolute cross sections for the reaction were determined by measuring the yield of Cu^{67} relative to the yield of the monitor reaction $\mathrm{Al}^{27}(p,3pn)\mathrm{Na}^{24}$. The values chosen for the cross section of the monitor reaction are given in Table I. The absolute disintegration rates were determined for Na^{24} and Cu^{67} by beta-gamma coincidence counting. The values for the cross sections of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction are given in Table I and the excitation function is shown in Fig. 1. The uncertainties listed are the standard deviations from the average for identical bombardments. The

TABLE I. Cross section of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction.

E	$\sigma\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$	$\sigma\mathrm{Al}^{27}(p,3pn)\mathrm{Na}^{24}$
80	11.8 ± 1.5	10.3[a]
130	18.1 ± 0.8	10.0[a]
210	14.6 ± 3.3	9.3[a]
250	20.2 ± 4.7	10.0[a]
300	21.3 ± 2.1	11.0[a]
350	25.8 ± 1.7	10.7[b]
400	20.8 ± 5.8	10.7[b]
430	24.9 ± 2.8	10.7[b]

[a] Value from H. G. Hicks, P. C. Stevenson, and W. E. Nervik, Phys. Rev. **102**, 1390 (1956).

[b] Value from J. B. Cumming, G. Friedlander, and C. E. Swartz, Phys. Rev. **111**, 1386 (1958).

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[14] D. J. Reuland, N. K. Ganguly, and A. A. Caretto, Jr., following paper, Phys. Rev. **133**, B1171 (1964).

[15] F. Exner, J. Am. Chem. Soc. **25**, 896 (1903).

[16] P. Kruger and N. Sugarman, Phys. Rev. **99**, 1459 (1959).

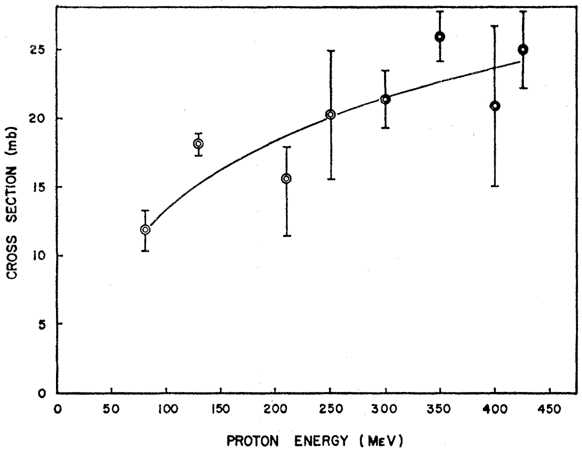

FIG. 1. Excitation function of the $Zn^{68}(p,2p)Cu^{67}$reaction.

decay scheme data for Na^{24} and Cu^{67} were taken from the compilation of Strominger *et al.*[17] Due to uncertainties in the measurements of the absolute disintegration rates, the determination of the chemical yields of the copper samples, and the resolution of the decay curves, the absolute cross sections are expected to be accurate to ±15% exclusive of the uncertainty in the value of the monitor cross sections.

In Table II are listed the effective recoil ranges determined in these experiments. These have been defined in a manner analogous to that used by Singh and Alexander,[12] and Sugarman *et al.*[18] The average component of the range in the forward direction is defined as the effective forward range, and is given by the product (FW), where F is the fraction of the total Cu^{67} activity observed in the forward catcher foil and W is the target thickness in μg per cm². The effective backward range is given by BW, and the effective perpendicular range is $2PW$, where P is the average fraction of Cu^{67} activity observed in the catchers exposed with the target plane at 10° to the beam direction. The values reported are the averages of at least two measurements and the uncertainties quoted are the standard deviation from the average. The last column in Table II gives the ratio F/B, the fraction of activity observed in the forward direction to the fraction observed in the backward direction. The dependence of the effective ranges on the proton bombarding energy is shown in Fig. 2 and the bombarding energy dependence of F/B in Fig. 3.

TABLE II. Effective recoil ranges.

Incident energy (MeV)	Effective forward range, FW μg/cm²	Effective backward range, BW μg/cm²	Effective perpendicular range, $2PW$ μg/cm²	Forward-backward ratio F/B
80	82.3±2.1	11.5±1.2		7.17±2.01
130	54.1±5.8	13.5±2.3	72.4±16.8	4.03±0.82
200	45.8±3.3	18.6±4.8	57.0± 5.2	2.46±0.61
250	53.1±5.3	18.9±4.3		2.81±0.70
300	42.7±9.1	14.2±1.3	66.8± 4.8	3.00±0.70
350	48.5±3.8	18.5		2.61±0.56
400	46.9±5.3	11.7±2.1	69.4± 5.4	4.01±0.84
430	42.6±0.6	16.0±6.4		2.66±1.10

The uncertainty in the values of the effective ranges due to straggling effects is large. Using the notation of Lindhard and Scharff,[19] all the effective ranges presented in Table II have uncertainties due to straggling of the order of 50% of the listed value of the average projected range.

DISCUSSION

The interpretation of the results of thick-target thick-catcher recoil studies investigated to date[20] has made use of various velocity vector diagrams designed to plausibly predict the product recoil behavior. In the use of such velocity vector diagrams common notation is that the target nucleus receives a velocity **v** which represents a component of the center of mass velocity arising possibly from the initial interaction, and the velocity **V**, which may or may not be isotropic in the moving target system. The velocity **V** has frequently been ascribed to any secondary affects of the interaction. In the case of the simple nuclear reactions, such as (p,pn) and ($p,2p$) in which the mechanism for the reaction is assumed, as a very first approximation, to involve a pure knock-out without any further manifestation of the interaction, the product nuclide is assumed to recoil with a momentum equal in magnitude

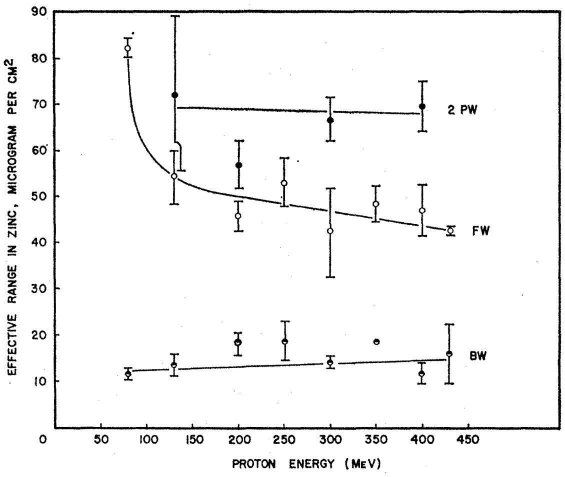

FIG. 2. Effective recoil ranges versus incident proton energy.

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[17] D. Strominger, J. M. Hollander, and G. T. Seaborg, Rev. Mod. Phys. **30**, 585 (1958).

[18] N. Sugarman, M. Campos, and K. Wielgoz, Phys. Rev. **101**, 388 (1956).

[19] J. Lindhard and M. Scharff, Phys. Rev. **124**, 128 (1961).

[20] B. G. Harvey, Ann. Rev. Nucl. Sci. **10**, 235 (1960).

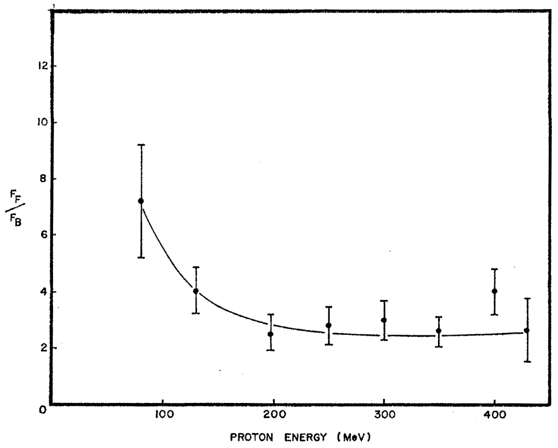

FIG. 3. Ratio of the effective forward range to the effective backward range versus incident proton energy.

to that of the struck nucleon prior to collision, but opposite in direction. This has occasionally been referred to as the "hole" momentum. The use of a velocity vector diagram to interpret the results of recoil studies of this type is of dubious value. This is because the quantity of interest is the determination of the momentum and angular distribution of the struck nucleon prior to collision by the measurement of the velocities or energies of the recoil nuclei in the laboratory system. In actuality, the spatial distribution of the recoil nucleus is related to the quantities of interest via the appropriate Fourier transformation. It is therefore not at all clear that a simple velocity vector diagram can be employed to obtain the desired nucleon momentum distribution. Furthermore, it is doubtlessly incorrect to assume that the incident and outgoing particles have no effect in the course of the interaction. Any nuclear momentum deposition produced by such effects as the interaction of the incident and outgoing particles with the nuclear potential, or from the absorption and refraction of the incident and outgoing particles in the nuclear potential, is intimately involved in the momentum deposition arising from the removal of a particular bound nucleon. Hence, it is essentially impossible to separate these effects physically or to attach physical significance in terms of a reaction mechanism to the two velocity vectors **v** and **V**. In addition to these fundamental reasons, other problems are encountered when a velocity vector diagram is employed to interpret recoil results. These are mainly: (1) only three experimental quantities are determined, but the most plausible velocity vector diagram requires at least four independent variables, (2) the angular distribution of the recoiling nuclei is usually assumed in the form $(a+b\cos^2\theta)$, whereas actually this distribution is one of the quantities that should be determined, and (3) in general the proposal of a velocity vector diagram requires an assumption as to the reaction mechanism, whereas again, this is one of the factors which is to be determined.

Mechanisms of (p,2 nucleon) Reactions

Several mechanisms have been proposed by which (p,2 nucleon) reactions take place at incident proton energies above 100 MeV.

(1) The clean knock-out mechanism involves an interaction of the incident proton with one of the target nucleons in which both collision partners promptly escape from the nucleus without further interaction and in which the residual nuclear excitation energy is less than about 10 MeV.

(2) A multiple collision knock-out mechanism may take place wherein the incident particle engages in more than one intranuclear nucleon-nucleon scattering collision followed by the prompt emergence of two nucleons and again the deposition of less than 10 MeV of nuclear excitation. No distinction will be made between these two mechanisms in the succeeding discussion. Whenever the knock-out mechanism is mentioned it will imply either mechanism (1) or (2) or both.

(3) A third possibility would involve the inelastic scattering of the incident particle with a nucleon in which a particle of the same identity emerges promptly, followed by the evaporation of the second nucleon on a slower time scale. The initial inelastic scattering would be required to deposit excitation energy between about 10–20 MeV. This inelastic scattering followed by nucleon evaporation can be abbreviated the ISE mechanism.

(4) The (p,pn) reaction could take place by reaction paths involving the emission of deuterons [i.e., the (p,d) reaction]; and (p,2 nucleon) reactions in general could take place via reaction paths involving mesons. Both of these mechanisms are considered unlikely in the 100–400 MeV region. The (p,d) reaction decreases very rapidly with increasing energy[21] and becomes insignificant at incident energies in excess of 100 MeV.[22] Since 400 MeV is only somewhat above the meson threshold, it is not expected that mesons play a major role in these reactions at the energies under consideration here.

Comparison of the $Zn^{68}(p,2p)Cu^{67}$ Reaction with Other "Simple" Nuclear Reaction Recoil Studies

The data in Table II indicates that the effective recoil ranges (FW, $2PW$, and BW) are forwardly peaked with sizeable transverse contributions at all the measured energies. Actually the effective transverse range is larger than the effective forward range at the measured energies. This behavior is similar to that

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[21] J. Heidmann, Phys. Rev. **80**, 171 (1950).

[22] W. Hess and B. Moyer, Phys. Rev. **101**, 337 (1956).

found for the $C^{12}(p,pn)C^{11}$ and $Cu^{65}(p,pn)Cu^{64}$ recoil studies.[11,12] The essentially energy independent effective ranges at energies above 200 MeV (Fig. 2) are also similar to the observed results of the $C^{12}(p,pn)C^{11}$ reaction, but quite different than the $Cu^{65}(p,pn)Cu^{64}$ reaction in which latter case a pronounced dip occurs for all three effective ranges at about 200–250 MeV. Merz and Caretto[11] interpreted the Cu^{64} recoil behavior resulting from the (p,pn) reaction in terms of a competition between the knock-out mechanism (mechanisms 1 and 2) and the ISE reaction mechanism (mechanism 3). The decrease in the forward effective range between about 100 and 200 MeV was related to the decrease in the probability of the ISE mechanism with increasing energy. The increase in the effective ranges at energies above 200–250 MeV was interpreted in terms of the increasing probability of the knock-out mechanism. In the case of the $C^{12}(p,pn)C^{11}$ reaction and the $Zn^{68}(p,2p)Cu^{67}$ reaction the ISE mechanism is less likely at all incident energies, than in the case of the $Cu^{65}(p,pn)Cu^{64}$ reaction due to the larger binding energies of the particular nucleon in question. Table III lists the energies required to evaporate protons, neutrons and alpha particles from the three nuclides after excitation by a (p,p') inelastic scattering. Note the lower binding energy of neutrons in Cu^{65} relative to the binding energy of neutrons in C^{12}. Note also that in the case of C^{12} and Zn^{68}, the evaporation of alphas or neutrons respectively would be preferential, but neither particle leads to the observed reaction product. Therefore the $Cu^{65}(p,pn)Cu^{64}$ reaction may involve a competition between the two most likely reaction mechanisms, (mechanism 1 or 2 and mechanism 3) while the $C^{12}(p,pn)C^{11}$ and $Zn^{68}(p,2p)Cu^{67}$ reactions may take place only by the knock-out mechanism.

TABLE III. Energies required to evaporate particles from target nuclei excited by (p,p') events.

Recoil study target nucleus	Neutron binding energy (MeV)	Proton binding energy plus effective potential barrier[a] (MeV)	Alpha binding energy plus effective potential barrier[a] (MeV)
C^{12}	18.32	18.84	13.49
Cu^{65}	10.01	17.83	28.13
Zn^{68}	9.99	20.39	28.06

[a] The effective potential barrier for the evaporation of protons and alphas from excited nuclei was assumed equal to the barrier calculated from classical electrostatics multiplied by a barrier penetrability factor. The factors used here are: 0.7 for protons, 0.83 for alpha particles. The nuclear radius was calculated by: $R=r_0[(A-m)^{1/3}+m^{1/3}]$, where $r_0=1.3\times10^{-13}$ cm, and m is the mass of the evaporation particle.

The ratio of the effective forward range to the effective backward range, FW/BW, for the three reactions investigated to date and at all energies above 100 MeV lies between about 2.5 and 5. If the recoil momentum is primarily due to the so-called "hole" momentum, then as a very first approximation, one might expect the recoil behavior to be isotropic in the laboratory system. Clearly this is not the case and one therefore seeks reasons, in terms of the reaction mechanism, for $FW/BW>1$. Several effects suggest themselves. Among these are: (1) If the number of collisions with nucleons moving toward the incident particle is different then the number of collisions with nucleons moving away from the incident particle, then the effective forward range would not be equal to the effective backward range. This effect is the only one of the succeeding list which is basically independent of all the other effects. Actually, it is the product of the cross section for nucleon-nucleon scattering at the particular center of mass energy in question, and the relative velocities of the colliding nucleons, which is proportional to the number of product recoils found in any given direction. (2) The incident and outgoing nucleons interact with the nuclear potential giving rise to a momentum deposition. Distortion effects of this type include the absorption or attenuation of the plane waves of the incident and outgoing particles, and refraction effects as long as the particle is in a region of the nucleus in which the nuclear potential is changing with distance. All such distortion effects are intimately involved in "selection" of the to-be-struck nucleon, and hence the resulting momentum deposition arises as an "inseparable" resultant from a combination of all these effects. (3) Depending on the momentum sharing of the two outgoing particles, the nuclear region in which successful (p,2 nucleon) reactions can take place varies.[23,24] In those cases where the outgoing particles share the incident momentum nearly equally, the successful reaction region lies in a surface zone on the side of the nucleus opposite from the point of entry of the incident proton. As the momentum sharing becomes far from equal, this region tends to move into an equatorial zone, at right angles to the incident particle direction. The localization of the successful reaction zone produces uncertainties in the momentum of the to-be-struck nucleon and also in its direction. This spatial effect in itself can account for $FW/BW>1$. In integral experiments, such as are reported here, the experiment averages the total number of recoils found in any given direction, which then precludes the assignment of an effective reaction zone in any simple way.

Velocity Angular Distribution

The effective ranges measured for the $Zn^{68}(p,2p)Cu^{67}$ reaction were converted into recoil ranges by assuming that the angular distribution of the Cu^{67} recoils could be described by a distribution in which the recoil velocity is expressed as a power series in the cosine of the scattering angle: $V=V_0(1+a\cos\theta+b\cos^2\theta)$. The choice of this angular distribution is related to the simple observation that such a distribution will reproduce the

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[23] J. R. Grover (private communication).

[24] P. A. Benioff, Phys. Rev. 119, 324 (1960).

experimental situation wherein there is a large forward-backward recoil ratio and in which there are sizeable numbers of transverse recoils. It is not the intention of these authors to make any physical conclusions from the results of calculations using this velocity distribution. Rather, what one hopes to accomplish by this process is: (1) to fit the data to the assumed velocity angular distribution, (2) to calculate the recoil ranges from this fit, and (3) to compare the recoil kinetic energies, as obtained from the recoil ranges, with the "hole" energy resulting from the knock-out mechanism, without making any detailed examination of the various velocity components that may arise as a result of the interaction. The purpose of this latter comparison is simply to provide a frame of reference as to the magnitude of the quantities involved.

Using the above velocity angular distribution, the following equations relating the fraction of recoils found in the forward, backward, and transverse directions can be derived assuming a simple one-step interaction:

$$F=\frac{R}{4W}\left(1+\frac{4}{3}a+\frac{a^2}{2}+\frac{4}{5}ab+b+\frac{b^2}{3}\right),$$

$$B=\frac{R}{4W}\left(1-\frac{4}{3}a+\frac{a^2}{2}-\frac{4}{5}ab+b+\frac{b^2}{3}\right),$$

$$P=\frac{R}{4W}\left(1+\frac{1}{4}a^2+\frac{1}{2}b+\frac{1}{8}b^2\right),$$

where F, B, and P are the fraction of the total number of recoils that are found in the forward, backward, and transverse directions, respectively, R is the recoil range, W the target thickness, both expressed in μg/cm^2, and a and b are the coefficients of the cosine power series. Ranges and the parameters a and b, calculated from these equations for the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction are listed in Table IV.

Three of the four sets of data predict ranges between 171 and 177 μg/cm^2. The low value at 200 MeV is a result of the low effective perpendicular range at this energy. There are reasons to suspect that this value may be low. Leaving out the data at 200 MeV, the average range between 130 and 400 MeV is 173.8 μg/cm^2 which corresponds to a recoil kinetic energy of about 800 keV.[25] The recoil kinetic energy of a Cu^{67} nucleus arising from the abrupt removal of a proton from the top of the nuclear well of Zn^{68} is about 610 keV[26] which corresponds to a range of about 150 μg/cm^2. The ranges calculated by this method are therefore at least consistent with what might be expected. The comparison which should be made, however, is these experimental results with the results from a quantal calculation in which the averaging of the recoil ranges due to the experimental geometry was also taken into account.

TABLE IV. Calculated recoil ranges of the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction.

Incident proton energy (MeV)	Cu^{67} recoil range (μg/cm^2)	Coefficients of cosine power series a	b
130	171.0	0.976	−1.058
200	128.9	0.992	−0.941
300	173.5	0.868	−1.195
400	177.0	0.841	−1.075

Conclusions

The recoil behavior of the product nucleus from the $\mathrm{Zn}^{68}(p,2p)\mathrm{Cu}^{67}$ reaction, as examined by the thick-target, thick-catcher experimental technique, is consistent with the following conclusions: (1) the most probable mechanism for the reaction is the knock-out mechanism at all the energies for which measurements were made, and (2) the experimental data can be fitted to a power cosine series to give recoil ranges in semiquantitative agreement with theoretical expectations.

ACKNOWLEDGMENTS

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[25] N. Porile (unpublished data).

[26] A. A. Ross, H. Mark, and R. D. Lawson, Phys. Rev. **102**, 1613 (1956).